

Exam Level Questions

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Chapter 1

The Straight Line

1D Exam Style Questions

1. $m = \frac{-7-5}{4-2}$

$m = -2$

perpendicular gradient $= \frac{1}{2}$

$a = 1, b = 3, m = \frac{1}{2}$

$y - 3 = \frac{1}{2}(x - 1)$

$2y - 6 = x - 1$

$x - 2y + 5 = 0$

2. $\text{Midpoint}_{PQ} = \left(\frac{-5+7}{2}, \frac{2+(-4)}{2} \right)$

$\text{Midpoint}_{PQ} = (1, -1)$

3. $5x - 2y + 3 = 0$

$2y = 5x + 3$

$y = \frac{5}{2}x + \frac{3}{2}$

parallel means same gradient $\therefore m = \frac{5}{2}$

$a = -2, b = 4, m = \frac{2}{5}$

$y - 4 = \frac{5}{2}(x + 2)$

$2y - 8 = 5x + 10$

$5x - 2y + 18 = 0$

4. Using $m = \tan \theta$, $m = \tan 150$

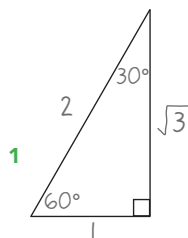
$\tan 150 = -\tan 30$

$= -\frac{1}{\sqrt{3}}$

$a = 2, b = -3, m = -\frac{1}{\sqrt{3}}$

$y + 3 = -\frac{1}{\sqrt{3}}(x - 2)$

$\sqrt{3}y + x = 2 - 3\sqrt{3}$



5. $m_{BC} = \frac{7-4}{5-3} = \frac{3}{2}$

$m_{AB} = \frac{4-(-2)}{3-k} = \frac{6}{3-k}$

since the points are collinear, $m_{AB} = m_{BC}$

$\frac{3}{2} = \frac{6}{3-k}$

$12 = 9 - 3k$

$k = -1$

6. $3x - 5y + 11 = 0$

$5y = 3x + 11$

$y = \frac{3}{5}x + \frac{11}{5}$

gradient of $L_1 = \frac{3}{5}$

perpendicular gradient $= -\frac{5}{3}$

$a = -4, b = 1$ and $m = -\frac{5}{3}$

$y - 1 = -\frac{5}{3}(x + 4)$

$5x + 3y + 17 = 0$

7. $2y - 3x = 5$ [A] $\times 4$

$3y + 4x = 16$ [B] $\times 3$

$8y - 12x = 20$ [C]

$9y + 12x = 48$ [D]

[C] + [D]

$17y = 68$

$y = 4$

$2(4) - 3x = 5$

$x = 1$

8. $\text{Midpoint}_{PQ} = \left(\frac{-2+6}{2}, \frac{4+2}{2} \right)$

$\text{Midpoint}_{PQ} = (2, 3)$

$m_{PQ} = \frac{4-2}{-2-6}$

$m_{PQ} = -\frac{1}{4}$

$m_{\text{PERP}} = 4$

$a = 2, b = 3, m = 4$

$$y - 3 = 4(x - 2)$$

$$y - 3 = 4x - 8$$

$$4x - y - 5 = 0$$

$$9. \text{Midpoint}_{AB} = \left(\frac{-3+7}{2}, \frac{5+3}{2} \right)$$

$$\text{Midpoint}_{AB} = (2, 4)$$

$$m_{CA} = \frac{-5-4}{1-2}$$

$$m_{CA} = 9$$

$$a = 2, b = 4, m = 9$$

$$y - 4 = 9(x - 2)$$

$$y - 4 = 9x - 18$$

$$9x - y - 14 = 0$$

$$10. m_{AC} = \frac{5 - (-5)}{(-3) - 1}$$

$$m_{AC} = -\frac{5}{2}$$

$$m_{PERP} = \frac{2}{5}$$

$$a = 7, b = 3 \text{ and } m = \frac{2}{5}$$

$$y - 3 = \frac{2}{5}(x - 3)$$

$$2x - 5y + 1 = 0$$

Chapter 2

Quadratic Functions

2D Exam Style Questions

$$1. \quad 5x^2 - 20x + 4$$

$$= (5x^2 - 20x) + 4$$

$$= 5(x^2 - 20x) + 4$$

$$= 5(x - 2)^2 + 4 - 5(-2)^2$$

$$= 5(x - 2)^2 + 4 - 20$$

$$= 5(x - 2)^2 - 16$$

$$2. \quad 2x^2 + 8x - 5$$

$$= (2x^2 + 8x) - 5$$

$$= 2(x^2 + 4x) - 5$$

$$= 2(x + 2)^2 - 5 - 2(2)^2$$

$$= 2(x + 2)^2 - 5 - 8$$

$$= 2(x + 2)^2 - 13$$

$$3. \quad -3x^2 - 12x + 8$$

$$= (-3x^2 - 12x) + 8$$

$$= -3(x^2 + 4x) + 8$$

$$= -3(x + 2)^2 + 8 - (-3)(2)^2$$

$$= -3(x + 2)^2 + 8 + 12$$

$$= -3(x + 2)^2 + 20$$

$$4. \quad -2x^2 + 4x - 9$$

$$= (-2x^2 + 4x) - 9$$

$$= -2(x^2 - 2x) - 9$$

$$= -2(x - 1)^2 - 9 - (-2)(-1)^2$$

$$= -2(x - 1)^2 - 9 + 2$$

$$= -2(x - 1)^2 - 7$$

$$5. \quad x^2 - 2x - 35 \leq 0$$

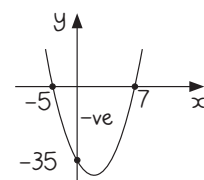
$$\text{let } x^2 - 2x - 35 = 0$$

$$(x - 7)(x + 5) = 0$$

$$x - 7 = 0, x + 5 = 0$$

$$x = 7, x = -5$$

$$\therefore x^2 - 2x - 35 \leq 0 \text{ when } -5 \leq x \leq 7$$



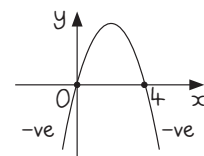
$$6. \quad 4x - x^2 < 0$$

$$\text{let } 4x - x^2 = 0$$

$$x(4 - x) = 0$$

$$x = 0, x = 4$$

$$\therefore 4x - x^2 < 0 \text{ when } x < 0 \text{ and } x > 4$$



$$7. \quad x^2 + (k + 9)x + 25 = 0$$

$$\text{for equal roots, } b^2 - 4ac = 0$$

$$a = 1, b = k + 9, c = 25$$

$$(k + 9)^2 - 4(1)(25) = 0$$

$$k^2 + 18k + 81 - 100 = 0$$

$$k^2 + 18k - 19 = 0$$

$$k + 19 = 0, k - 1 = 0$$

$$k = -19, k = 1$$

$$8. \quad x^2 + (k + 3)x + (5 - k) = 0$$

$$\text{for two real and distinct roots, } b^2 - 4ac > 0$$

$$\text{let } b^2 - 4ac = 0$$

$$a = 1, b = k + 3, c = 5 - k$$

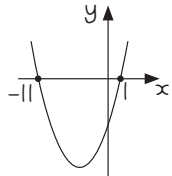
$$(k + 3)^2 - 4(1)(5 - k) = 0$$

$$k^2 + 6k + 9 - 20 + 4k = 0$$

$$k^2 + 10k - 11 = 0$$

$$(k + 11)(k - 1) = 0$$

$$k = -11, k = 1$$



$$\therefore k^2 + 10k - 11 > 0 \text{ when } k < -11 \text{ and } k > 1$$

$$9. \quad 2x^2 - 4x + (1 - p) = 0$$

for no real roots, $b^2 - 4ac < 0$

$$\text{let } b^2 - 4ac = 0$$

$$a = 2, b = -4, c = (1 - p)$$

$$(-4)^2 - 4(2)(1 - p) = 0$$

$$16 - 8 + 8p = 0$$

$$8 + 8p = 0$$

$$p = -1$$

$$\therefore p < -1$$

$$10. \quad mx^2 + (m - 3)x + 1 = 0$$

for two real and distinct roots, $b^2 - 4ac > 0$

$$\text{let } b^2 - 4ac = 0$$

$$a = m, b = m - 3, c = 1$$

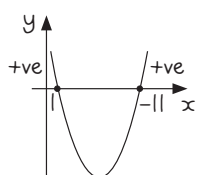
$$(m - 3)^2 - 4(m)(1) = 0$$

$$m^2 - 6m + 9 - 4m = 0$$

$$m^2 - 10m + 9 = 0$$

$$(m - 9)(m - 1) = 0$$

$$m = 9, m = 1$$



$$\therefore m^2 - 10m + 9 > 0 \text{ when } m < 1 \text{ and } m > 9$$

Chapter 3

The Circle

3D Exam Style Questions

$$1. \quad x^2 + y^2 - 8x + 46y - 480 = 0$$

$$2g = -8 \text{ and } 2f = 46$$

$$g = -4 \text{ and } f = 23$$

$$\text{circle centre} = (-g, -f)$$

$$\text{circle centre} = (4, -23)$$

$$A(4, -23) \quad B(-3, -19)$$

$$\text{Gradient of radius} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-19 - (-23)}{-3 - 4}$$

$$= -\frac{4}{7}$$

$$\text{Using } m_{\text{rad}} \times m_{\text{tan}} = -1$$

$$m_{\text{tan}} = \frac{7}{4}$$

$$a = -3, b = -19, m = \frac{7}{4}$$

$$y - b = m(x - a)$$

$$y - 19 = \frac{7}{4}(x + 3)$$

$$4y = 7x - 55$$

$$2. \quad x^2 + y^2 + 10x - 2y + p = 0$$

$$g = 5, f = -1, c = p$$

for a circle to exist,

$$\sqrt{g^2 + f^2 - c} > 0$$

$$\sqrt{5^2 + (-1)^2 - p} > 0$$

$$26 - p > 0$$

$$p < 26$$

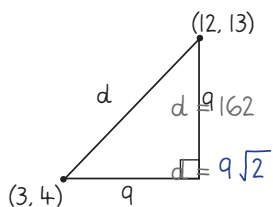
$$3. \quad \text{Circle: } (x - 3)^2 + (y - 4)^2 = 18$$

$$\text{Centre, } (3, 4)$$

$$\text{Radius, } = 3\sqrt{2}$$

$$\text{distance: } d^2 = q^2 + q^2$$

$$d^2 = 162$$



$$r_2 = \text{distance} - r_1 = q\sqrt{2} - 3\sqrt{2} = 6\sqrt{2}$$

$$a = 12, b = 13, r = 6\sqrt{2}$$

$$\text{circle}_2 = (x - 12)^2 + (y - 13)^2 = 72$$

4. Sub line $y = -2x + 4$ into the circle

$$(x - 5)^2 + (-2x + 4 - 14)^2 = 80$$

$$(x - 5)^2 + (-2x - 10)^2 = 80$$

$$x^2 - 10x + 25 + 4x^2 + 40x + 100 = 80$$

$$5x^2 + 30x + 45 = 0$$

$$5(x^2 + 6x + 9) = 0$$

$$5(x + 3)(x + 3) = 0$$

As line is tangent there is only one point of contact,

$$\therefore x = -3$$

$$\text{when } x = -3, y = -2(-3) + 4 = 10$$

$$\text{point of contact} = (-3, 10)$$

5. Sub line $y = x + 5$ into the circle

$$x^2 + (x + 5 - 3)^2 = 20$$

$$x^2 + (x + 2)^2 = 20$$

$$x^2 + x^2 + 4x + 4 = 20$$

$$2x^2 + 4x - 16 = 0$$

$$2(x^2 + 2x - 8) = 0$$

$$2(x + 4)(x - 2) = 0$$

$$x + 4 = 0 \text{ and } x - 2 = 0$$

$$\therefore x = -4 \text{ and } x = 2$$

$$\text{when } x = -4, y = (-4) + 5 = 1$$

$$\text{when } x = 2, y = (2) + 5 = 7$$

$$\text{points of contact} = (-4, 1) \text{ and } (2, 7)$$

6. At point A, $x = 0$

$$\therefore (0 - 3)^2 + (y - 1)^2 = 73$$

$$9 + y^2 - 2y + 1 - 73 = 0$$

$$y^2 - 2y - 63 = 0$$

$$(y - 9)(y + 7) = 0$$

$$\therefore A(0, -7)$$

$$\begin{matrix} (0, -7) & (3, 1) \\ x_1, y_1 & x_2, y_2 \end{matrix}$$

$$\text{Gradient of radius} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - (-7)}{3 - 0}$$

$$= \frac{8}{3}$$

$$\text{Using } m_{\text{rad}} \times m_{\text{tan}} = -1$$

$$m_{\text{tan}} = -\frac{3}{8}$$

$$a = 0, b = -7, m = -\frac{3}{8}$$

$$y - b = m(x - a)$$

$$y + 7 = -\frac{3}{8}(x + 0)$$

$$y = -\frac{3}{8}x - 7 \text{ or } 8x + 3y + 56 = 0$$

7. Centre₁ $(-7, -2)$, Radius₁ $= \sqrt{41} = 6.403...$

$$\text{Centre}_2 (4, 3), \text{Radius}_2 = \sqrt{20} = 4.472...$$

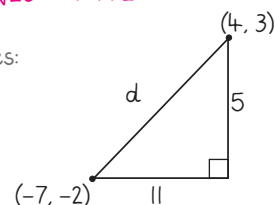
distance between centres:

$$d^2 = 11^2 + 5^2$$

$$d = 12.083$$

$$r_1 + r_2 = 6.4 + 4.5 = 10.9$$

$\therefore$ since $10.9 < 12.1$, the circles DO NOT intersect



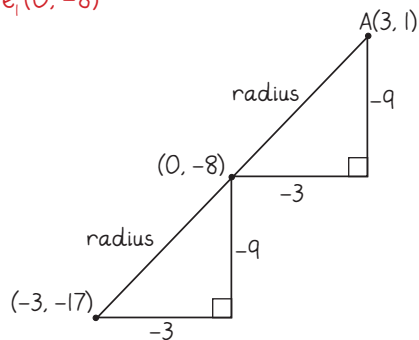
8. Centre₁ $(7, 6)$, Radius₁ $= 5$

For circle₂ x-value of centre $7 + 2 \times 5 = 17$

$$y\text{-value of centre } 6 + 4 \times 5 = 26$$

$$\therefore \text{equation} = (x - 17)^2 + (y - 26)^2 = 25$$

9. Centre, (0, -8)



x-value at B = -3 and y-value = -17

$$\therefore B(-3, -17)$$

10. Sub line, $y = 2x + p$ into circle

$$x^2 + (2x + p)^2 - 10x - 14(2x + p) - 6 = 0$$

$$x^2 + 4x^2 + 4xp + p^2 - 10x - 28x - 14p - 6 = 0$$

$$5x^2 + 4xp - 38x + p^2 - 14p - 6 = 0$$

$$5x^2 + x(4p - 38) + (p^2 - 14p - 6) = 0$$

$$a = 5, b = 4p - 38, c = p^2 - 14p - 6$$

if line is tangent, $b^2 - 4ac = 0$

$$\therefore (4p - 38)^2 - 4(5)(p^2 - 14p - 6) = 0$$

$$p^2 + 6p - 391 = 0$$

$$(p + 23)(p - 17) = 0$$

$$p = -23 \text{ and } p = 17$$

Chapter 4

Functions

4D Exam Style Questions

1. $g(x) = (x + 3)^3$

let $y = (x + 3)^3$

$$\sqrt[3]{y} = x + 3$$

$$\sqrt[3]{y} - 3 = x$$

$$g^{-1}(x) = \sqrt[3]{x} - 3$$

2. $k(x) = 5x^3 + 2$

let $y = 5x^3 + 2$

$$y - 2 = 5x^3$$

$$\frac{y - 2}{5} = x^3$$

$$\sqrt[3]{\frac{y - 2}{5}} = x$$

$$f^{-1}(x) = \sqrt[3]{\frac{x - 2}{5}}$$

3. $f(x) = \frac{3}{x} - 4$

let $y = \frac{3}{x} - 4$

$$y + 4 = \frac{3}{x}$$

$$x(y + 4) = 3$$

$$x = \frac{3}{y + 4}$$

$$f^{-1}(x) = \frac{3}{x + 4}$$

4. $k(x) = f(g(x))$ when $f(x) = x^2 - 2$ and $g(x) = 5 - x^3$

$$= f(5 - x^3)$$

$$= (5 - x^3)^2 - 2$$

5. $f(x) = x^2 - 2$ and $g(x) = 5 - x^3$

$$f(g(x)) = f(5 - x^3)$$

$$= 3(5 - x^3)^2 - 2$$

6. $f(x) = 2x^2 - 1$ and $g(x) = 4x + 1$

$$f(g(x)) = f(4x + 1)$$

$$= 2(4x + 1)^2 - 1$$

7. $f(x) = \frac{1}{(x + 1)(x - 2)}$ and $g(x) = x + 3$

$$f(g(x)) = f(x + 3)$$

$$= \frac{1}{((x + 3) + 1)((x + 3) - 2)}$$

$$= \frac{1}{(x + 4)(x + 1)}$$

$$\text{undefined when } (x + 4)(x + 1) = 0$$

$$\text{undefined when } x = -4 \text{ and } x = -1$$

8. $f(x) = 3x^2 - 4$ and $g(x) = x - 2$

$$g(f(x)) = g(3x^2 - 4)$$

$$= 3x^2 - 4 - 2$$

$$= 3x^2 - 6$$

as 1 is minimum value $\therefore$ range $g(f(x)) \geq -6$

9. $h(x) = f(g(x))$ when $f(x) = 2x - 5$ and $g(x) = x^2 + 1$

$$= g(2x - 5)$$

$$= (2x - 5)^2 + 1$$

as 1 is minimum value $\therefore$ range $h(x) \geq 1$

10. $f(x) = \sqrt{2x + 1}$ and $g(x) = 3 - x$

$$f(g(x)) = f(3 - x)$$

$$= \sqrt{2(3 - x) + 1}$$

$$= \sqrt{6 - 2x + 1}$$

$$= \sqrt{7 - 2x}$$

$$7 - 2x \geq 0$$

$$7 \geq 2x$$

$$\frac{7}{2} \geq x$$

$\therefore$ undefined when $\frac{7}{2} < x$

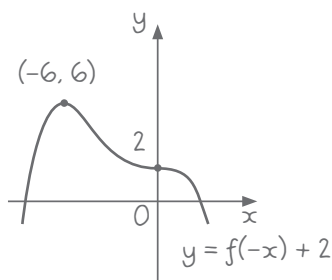
Chapter 5

Graphs of Related Functions

5D Exam Style Questions

1. Reflection in y-axis identifiable from graph

Vertical translation of '+2' identifiable from graph

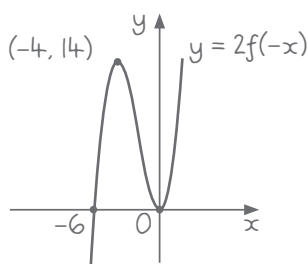


2. Original graph shifted $x + 4$, $y + 2$:

$$x = 6 \text{ and } y = 2$$

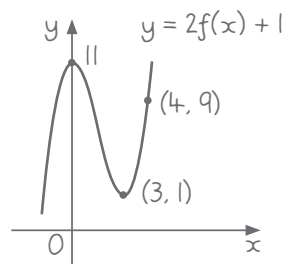
3. Reflection in y-axis identifiable from graph

Apply appropriate vertical scaling identifiable from graph



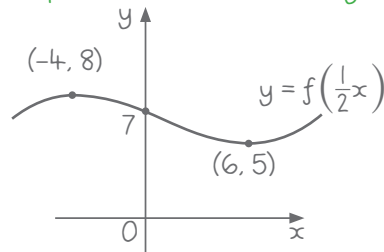
4. Apply appropriate vertical scaling identifiable from graph

Vertical translation of '+1' identifiable from graph



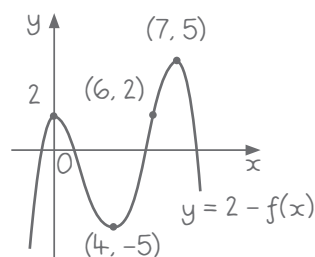
5. Apply appropriate horizontal scaling identifiable from graph

All points annotated correctly on graph



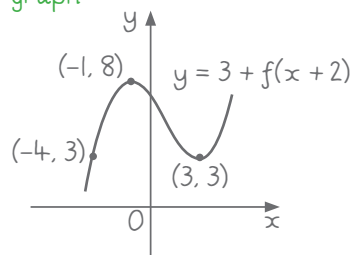
6. Reflection in x-axis identifiable from graph

Vertical translation of '+2' identifiable from graph



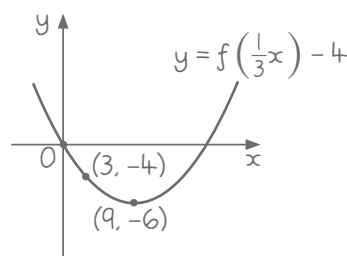
7. Horizontal translation of '-2' identifiable from graph

Vertical translation of '+3' identifiable from graph

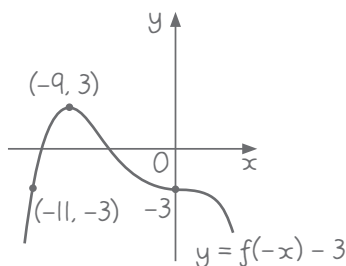


8. Apply appropriate horizontal scaling identifiable from graph

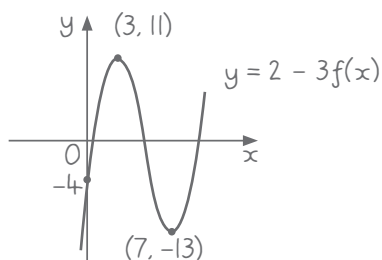
Vertical translation of '-4' identifiable from graph



9. Reflection in y-axis identifiable from graph 1
Vertical translation of '-3' identifiable from graph 1



10. Reflection in x-axis identifiable from graph 1
Apply appropriate vertical scaling identifiable from graph 1
Vertical translation of '+2' identifiable from graph 1



Chapter 6

Recurrence Relations

6D Exam Style Questions

1. $u_{n+1} = 0.8u_n + 16$
for a limit to exist, $-1 < a < 1$
 $-1 < 0.8 < 1 \therefore$ limit exists
 $L = \frac{b}{1-a}$
 $L = \frac{16}{1-0.8}$ 1
 $L = 80$ 1
2. $u_{n+1} = 0.6u_n + 3$ and $u_0 = 3$
 $u_1 = 0.6u_0 + 3$
 $u_1 = 0.6(3) + 3 = 4.8$ 1
 $u_2 = 0.6(4.8) + 3 = 5.88$
 $u_3 = 0.6(5.88) + 3 = 6.528$
 $u_4 = 0.6(6.528) + 3 = 6.9168$
 $u_5 = 0.6(6.9168) + 3 = 7.15008$ 1
the smallest value of $n = 5$ for which $u_n > 7$ 1

3. $u_{n+1} = 0.95u_n + 8$ and $u_0 = 150$ 1
for a limit to exist,
 $-1 < a < 1$
 $-1 < 0.95 < 1 \therefore$ limit exists
 $L = \frac{b}{1-a}$
 $L = \frac{8}{1-0.95}$ 1
 $L = 160$
The long-term population of sheep will settle around 160 1
4. let $u_0 = 7$, $u_1 = -25$ and $u_2 = 71$
 $u_1 = au_0 + b$
 $u_2 = au_1 + b$
 $-25 = 7a + b$ [A]
 $71 = -25a + b$ [B] [A] - [B] 1
 $-96 = 32a$
 $a = -3$ 1
when $a = -3$, $b = -4$ 1
5. $u_{n+1} = ku_n + 15$, using limit formula
 $L = \frac{b}{1-a}$
 $25 = \frac{15}{1-k}$ 1
 $25(1-k) = 15$ 1
 $25 - 25k = 15$
 $25k = 10$
 $k = \frac{2}{5}$ or 0.4 1
6. $u_{n+1} = 4u_n + 5$ where $u_2 = 9$
 $u_2 = 4u_1 + 5$
 $9 = 4u_1 + 5$ 1
 $u_1 = 1$ 1
 $u_1 = 4u_0 + 5$
 $1 = 4u_0 + 5$
 $u_0 = -1$ 1

7. $u_{n+1} = 60 - 0.5u_n$ where $u_0 = 4$

for a limit to exist, $-1 < a < 1$

$-1 < -0.5 < 1 \therefore$ limit exists

$$L = \frac{b}{1-a}$$

$$L = \frac{60}{1+0.5}$$

$$L = 40$$

8. As both relations approach the same limit,

$$\frac{b}{1-a} = \frac{b}{1-a}$$

$$\frac{p}{1-0.3} = \frac{q}{1-0.9}$$

$$\frac{p}{0.7} = \frac{q}{0.1}$$

$$0.1p = 0.7q$$

$$p = 7q$$

9. As both relations approach the same limit,

$$\frac{b}{1-a} = \frac{b}{1-a}$$

$$\frac{20}{1-a} = \frac{38}{1-a^2}$$

$$20(1-a^2) = 38(1-a)$$

$$20 - 20a^2 = 38 - 38a$$

$$20a^2 - 38a + 18 = 0$$

$$2(10a^2 - 19a + 9) = 0$$

$$2(10a - 9)(a - 1) = 0$$

$$a = \frac{9}{10} \text{ and } a = 1$$

for a limit to exist, $-1 < a < 1 \therefore a$ can only be $\frac{9}{10}$

$$L = \frac{b}{1-a} = \frac{20}{1-0.9} = 200$$

10. $u_{n+1} = pu_n + 3$ where $u_0 = 20$ and $u_2 = 7.4$

$$u_1 = pu_0 + 3$$

$$u_1 = 20p + 3$$

$$u_2 = pu_1 + 3$$

$$7.4 = p(20p + 3) + 3$$

$$7.4 = 20p^2 + 3p + 3$$

$$0 = 20p^2 + 3p - 4.4$$

using quadratic root formula to solve,

$$p = \frac{-3 \pm \sqrt{3^2 - 4(20)(-4.4)}}{2(20)}$$

$$p = -\frac{11}{20} \text{ and } \frac{2}{5}$$

Chapter 7

Polynomials

7D Exam Style Questions

1.
$$\begin{array}{r|rrrr} 2 & 2 & -3 & -3 & 2 \\ 1 & & & & \end{array}$$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -3 & 2 \\ & & 4 & 2 & -2 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

since remainder = 0, $(x - 2)$ is a factor

$$(x - 2)(2x^2 + x - 1) = 0$$

$$(x - 2)(2x - 1)(x + 1) = 0$$

$$x = 2, x = \frac{1}{2}, x = -1$$

2.
$$\begin{array}{r|rrrr} -1 & 1 & a & b & -4 \\ & & -1 & 1-a & a-b-1 \\ \hline & 1 & a-1 & 1-a-b & a-b-5=0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & a & b & -4 \\ & & 2 & 2a+4 & 4a+8+2b \\ \hline & 1 & 2 & 2a+4+b & 4a+4+2b=6 \end{array}$$

$$a - b = 5 \quad [A] \times 2$$

$$4a + 2b = 2 \quad [B]$$

$$2a - 2b = 10 \quad [C]$$

$$6a = 12$$

$$a = 2$$

$$2 - b = 5$$

$$b = -3$$

3. curves are equal when,

$$x^3 - 3x^2 + 2x + 4 = 2x + 4$$

$$x^3 - 3x^2 = 0$$

$$x^2(x - 3) = 0$$

$$x = 0, x = 3$$

$$\text{when } x = 0, y = 2(0) + 4 = 4$$

$$\text{when } x = 3, y = 2(3) + 4 = 10$$

points of intersection: (0, 4) and (3, 10)

$$4. \begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

since remainder = 0, $x = 1$ is a root of $f(x)$

$$(x - 1)(x^2 - 5x + 6) = 0$$

$$(x - 1)(x - 3)(x - 2) = 0$$

$$x = 1, x = 3, x = 2$$

$$5. \begin{array}{r|rrrr} -3 & 1 & 2 & 0 & -4 & 1 \\ \hline \end{array}$$

$$\begin{array}{r|rrrrr} -3 & 1 & 2 & 0 & -4 & 1 \\ & & 1 & -3 & 3 & -9 & 39 \\ \hline & 1 & -1 & 3 & -13 & 40 \end{array}$$

remainder = 40

$$6. \begin{array}{r|rrrr} -1 & 1 & -1 & 3 & 5 \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 3 & 5 \\ & & -1 & 2 & -5 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

since $f(-1) = 0$, $(x + 1)$ is a factor

$$(x + 1)(x^2 - 2x + 5) = 0$$

sub $a = 1$, $b = -2$, $c = 5$ into $b^2 - 4ac$

$$(-2)^2 - 4(1)(5) = -16$$

since $b^2 - 4ac < 0$, the quadratic has no real roots,

$x = -1$ is the only real root

$$7. \begin{array}{r|rrrr} -1 & 3 & 1 & -3 & -1 \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} -1 & 3 & 1 & -3 & -1 \\ & & -3 & 2 & 1 \\ \hline & 3 & -2 & -1 & 0 \end{array}$$

since remainder = 0, $(x + 1)$ is a factor

$$(x + 1)(3x^2 - 2x - 1)$$

$$(x + 1)(3x + 1)(x - 1)$$

$$8. y = k(x + 2)(x - 1)(x - 1)$$

$$\text{when } x = 0, y = -6$$

$$-6 = k(0 + 2)(0 - 1)(0 - 1)$$

$$-6 = 2k$$

$$k = -3$$

$$y = -3(x + 2)(x - 1)^2$$

$$9. f(x) = k(x)(x - 3)(x - 5)(x - 5)$$

$$\text{when } x = 1, y = -8$$

$$-8 = k(1)(1 - 3)(1 - 5)(1 - 5)$$

$$-8 = -32k$$

$$k = \frac{1}{4}$$

$$f(x) = \frac{1}{4}x(x - 3)(x - 5)^2$$

$$\begin{array}{r}
 10. \quad \begin{array}{rrrr}
 2 & p & q & 6 \\
 \hline
 & 6 & 3p+18 & 9p+54+3q \\
 \hline
 1 & p+6 & 3p+18+q & 9p+60+3q=0
 \end{array} \\
 \\
 \begin{array}{rrrr}
 -1 \mid 2 & p & q & 6 \\
 \hline
 & -2 & 2-p & p-2-q \\
 \hline
 1 & p-2 & 2-p+q & p+4-q=16
 \end{array}
 \end{array}$$

$$9p + 3q = -60 \text{ [A]} \div 3$$

$$p - q = 12 \text{ [B]}$$

$$3p + q = -20 \text{ [C]}$$

$$\begin{array}{r}
 3p + q = -20 \text{ [C]} \\
 \hline
 4p = -8
 \end{array}
 \quad \begin{array}{l}
 \text{[C]} + \text{[B]}
 \end{array}$$

$$p = -2$$

$$9(-2) + 3q = -60$$

$$q = -14$$

$$f(x) = (x+1)(2x^2+4x-2)$$

$$f(x) = 2(x+1)(x^2+2x-1)$$

Chapter 8

Trigonometric Functions

8D Exam Style Questions

$$1. \quad 2\cos(3x - 30)^\circ = -1$$

$$\cos(3x - 30)^\circ = -\frac{1}{2}$$

$$\text{Acute angle: } \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$$3x - 30 = 120, 240$$

$$3x = 150, 270$$

$$x = 50^\circ, 90^\circ, 170^\circ$$

$$2. \quad \sqrt{35}\sin(a + 0.23) = 4.8$$

$$\sin(a + 0.23) = \frac{4.8}{\sqrt{35}}$$

$$\text{Acute angle: } \sin^{-1}\left(\frac{4.8}{\sqrt{35}}\right) = 0.946\dots$$

$$a + 0.23 = 0.946\dots, 2.195\dots$$

$$a = 0.72, 1.97$$

$$3. \quad a = -\frac{1}{2}(8 + (-4)) = -6$$

$$b = \frac{1}{2}(8 + (-4)) = 2$$

$$4. \quad g\left(\frac{\pi}{12}\right) = 3\left(\frac{\pi}{12}\right) = \frac{\pi}{4}$$

$$f\left(\frac{\pi}{4}\right)$$

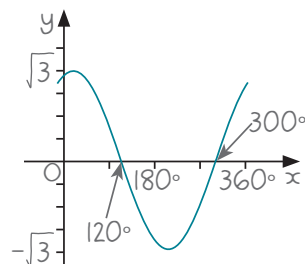
$$= 4\cos\frac{\pi}{4}$$

$$= 4\left(\frac{1}{\sqrt{2}}\right)$$

$$= 2\sqrt{2}$$

$$a = 2$$

5.



two roots: $(120^\circ, 0)$ and $(300^\circ, 0)$

two turning points $(30^\circ, \sqrt{3})$ and $(210^\circ, -\sqrt{3})$

y-intercept and value of y at 360° from graph

$$6. \quad 4\sin\left(x - \frac{\pi}{6}\right) + 6 = 7$$

$$4\sin\left(x - \frac{\pi}{6}\right) = 1$$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{4}$$

$$\text{Acute angle: } \sin^{-1}\left(\frac{1}{4}\right) = 0.252\dots$$

$$x - \frac{\pi}{6} = 0.252\dots, 3.412\dots$$

$$x_0 = 0.78, x_r = 3.41$$

$$7. \quad f\left(\frac{\pi}{3}\right) = 3\sin\left(2\left(\frac{\pi}{3}\right) + \left(\frac{\pi}{6}\right)\right)$$

$$= 3\sin\frac{5\pi}{6}$$

$$= 3\left(\frac{1}{2}\right)$$

$$= \frac{3}{2}$$

$$8. \quad f(g(x)) = f(2x + 60)$$

$$\sqrt{3}\tan(2x + 60)^\circ - 1 = 0$$

$$\sqrt{3}\tan(2x + 60)^\circ = 1$$

$$\tan(2x + 60)^\circ = \frac{1}{\sqrt{3}}$$

$$\text{Acute angle} = 30^\circ$$

$$2x + 60 = 30, 210$$

$$2x = -30, 150$$

$$x = -15, 75$$

$$x^\circ = 75^\circ, 165^\circ, 255^\circ, 345^\circ$$

9. $q(t) = 2\cos(3\pi t)$

$$q'(t) = -2\sin(3\pi t) \times 3\pi$$

$$q'(0.5) = -6\pi \times \sin\left(\frac{3\pi}{2}\right)$$

$$= -6\pi(-1)$$

$$= 6\pi$$

10. $y = \sin 2x + 1$

$$\frac{dy}{dx} = \cos 2x \times 2$$

$$2 \cos 2x = 1$$

$$\cos 2x = \frac{1}{2}$$

$$2x = 60$$

$$x = 30^\circ$$

Chapter 9

Addition Formulae

9D Exam Style Questions

1. $\sqrt{7^2 + 1^2} = \sqrt{50}$

$$\sin x = \frac{1}{\sqrt{50}} \text{ or } \cos x = \frac{7}{\sqrt{50}}$$

using $\cos 2x = 2\cos^2 x - 1$ or equivalent,

$$\cos 2x = 2 \times \frac{7}{\sqrt{50}} \times \frac{7}{\sqrt{50}} - 1$$

$$\cos 2x = 2 \times \frac{49}{50} - 1$$

$$\cos 2x = \frac{24}{25}$$

2. $\sqrt{1^2 + 1^2} = \sqrt{2}$

$$\sin q = \frac{1}{\sqrt{2}}, \cos q = \frac{1}{\sqrt{2}}$$

$$\sqrt{3^2 + 1^2} = \sqrt{8}$$

$$\sin r = \frac{1}{3}, \cos r = \frac{\sqrt{8}}{3}$$

$$\cos(q + r) = \cos q \cos r - \sin q \sin r$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{8}}{3} \times \frac{1}{\sqrt{2}} \times \frac{1}{3}$$

$$= \frac{2\sqrt{2} - 1}{3\sqrt{2}}$$

3. $\sin 2x + \sqrt{3} \cos x = 0$

$$2\sin x \cos x + \sqrt{3} \cos x = 0$$

$$\cos x(2\sin x + \sqrt{3}) = 0$$

$$\cos x = 0$$

$$2\sin x + \sqrt{3} = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{4\pi}{3}, \frac{5\pi}{3}$$

4. $f(x) = 2x$ and $g(x) = 2\cos x + 1$

$$f(g(x)) = f(2\cos x + 1) \quad g(f(x)) = g(2x)$$

$$= 2(2\cos x + 1)$$

$$= 2\cos 2x + 1$$

$$f(g(x)) = g(f(x))$$

$$2(2\cos x + 1) = 2\cos 2x + 1$$

$$4\cos x + 2 = 2\cos 2x + 1$$

$$2\cos 2x - 4\cos x - 1 = 0$$

$$2(2\cos^2 x - 1) - 4\cos x - 1 = 0$$

$$4\cos^2 x - 4\cos x - 3 = 0$$

$$(2\cos x + 1)(2\cos x - 3) = 0$$

$$2\cos x + 1 = 0$$

$$2\cos x - 3 = 0$$

$$\cos x = -\frac{1}{2}$$

$$\cos x = \frac{3}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

no solutions

5. $4\sin^2 x = 5\cos x + 5$

$$4(1 - \cos^2 x) = 5\cos x + 5$$

$$4\cos^2 x + 5\cos x + 1 = 0$$

$$(4\cos x + 1)(\cos x + 1) = 0$$

$$4\cos x + 1 = 0$$

$$\cos x + 1 = 0$$

$$\cos x = -\frac{1}{4}$$

$$\cos x = -1$$

$$x = 104.5^\circ, 255.5^\circ$$

$$x = 180^\circ$$

6. $\tan x - \sin^2 x \tan x = \sin x \cos x$

$$\tan x(1 - \sin^2 x)$$

$$\frac{\sin x}{\cos x}(\cos^2 x)$$

$$\sin x \cos x$$

7. $\sqrt{5^2 - 4^2} = 3$

$$\sin y = \frac{4}{5}, \cos y = \frac{3}{5}$$

$$\sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\sin x = \frac{3}{\sqrt{13}}, \cos x = \frac{2}{\sqrt{13}}$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$= \frac{3}{\sqrt{13}} \times \frac{3}{5} + \frac{2}{\sqrt{13}} \times \frac{4}{5}$$

$$= \frac{17}{5\sqrt{13}}$$

$$8. \cos 2x = \frac{3}{5}$$

$$\text{rearranging } \cos 2x = 2\cos^2 x - 1,$$

$$\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2} \text{ (start to rearrange)}$$

$$\cos^2 x = \frac{1}{2} \times \frac{3}{5} + \frac{1}{2}$$

$$\cos^2 x = \frac{4}{5}$$

$$\cos x = \frac{2}{\sqrt{5}}$$

$$9. \text{ shorter side for triangle } q = \sqrt{2}$$

$$\sin q = \frac{1}{\sqrt{3}}, \cos q = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\cos 2q = 2\cos^2 q - 1$$

$$= 2 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}} - 1$$

$$= \frac{1}{3}$$

$$\sin 2q = 2\sin q \cos q$$

$$= 2 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\sin r = \frac{2}{\sqrt{5}}, \cos r = \frac{1}{\sqrt{5}}$$

$$\cos(2q-r) = \cos 2q \cos r + \sin 2q \sin r$$

$$= \frac{1}{3} \times \frac{1}{\sqrt{5}} + \frac{2\sqrt{2}}{3} \times \frac{2}{\sqrt{5}}$$

$$= \frac{1+4\sqrt{2}}{3\sqrt{5}}$$

$$10. \text{ let } \cos 2x = 1 - \sin x$$

$$1 - 2\sin^2 x = 1 - \sin x$$

$$2\sin^2 x - \sin x = 0$$

$$\sin x(2\sin x - 1) = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

$$2\sin x - 1 = 0$$

$$x_p = \frac{\pi}{6}, x_0 = \frac{5\pi}{6}$$

Chapter 10

The Wave Function

10D Exam Style Questions

$$1. \text{ Let } 8\cos x + 5\sin x = k\cos(x-a)^\circ$$

$$8\cos x + 5\sin x = k(\cos x \cos a + \sin x \sin a)$$

$$8\cos x + 5\sin x = k\cos a \cos x + k\sin a \sin x$$

$$k\cos a = 8 \text{ and } k\sin a = 5$$

$$k = \sqrt{8^2 + 5^2}$$

$$k = \sqrt{89}$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{5}{8}$$

$$a = \tan^{-1}\left(\frac{5}{8}\right)$$

$$a = 32^\circ$$

$$\therefore 8\cos x + 5\sin x = \sqrt{89}\cos(x-32)^\circ$$

$$\text{where } 8\cos x + 5\sin x = 4$$

$$\text{let } \sqrt{89}\cos(x-32)^\circ = 4$$

$$\cos(x-32)^\circ = \frac{4}{\sqrt{89}}$$

$$x-32 = 64.9, 360-64.9$$

$$x-32 = 64.9, 295.1$$

$$x = 96.9^\circ, 327.1^\circ$$

$$2. \text{ Let } \sqrt{5}\sin x - 2\cos x = k\sin(x-a)^\circ$$

$$\sqrt{5}\sin x - 2\cos x = k(\sin x \cos a - \cos x \sin a)$$

$$\sqrt{5}\sin x - 2\cos x = k\cos a \sin x - k\sin a \cos x$$

$$k\cos a = \sqrt{5} \text{ and } k\sin a = 2$$

$$k = \sqrt{5+4}$$

$$k = 3$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{2}{\sqrt{5}} = 0.8944\dots$$

$$a = \tan^{-1}(0.8944\dots)$$

$$a = 41.8^\circ$$

$$\therefore \sqrt{5}\sin x - 2\cos x = 3\sin(x-41.8)^\circ$$

$$\text{where } \sqrt{5}\sin x - 2\cos x = 1$$

$$\text{let } 3\sin(x-41.8)^\circ = 1$$

$$\sin(x-41.8)^\circ = \frac{1}{3}$$

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$$x - 41.8 = 19.5, 180 - 19.5$$

$$x - 41.8 = 19.5, 160.5$$

$$x = 61.3^\circ, 202.3^\circ$$

3. Let $3 \cos x - \sin x = k \cos(x + a)$

$$3 \cos x - \sin x = k(\cos x \cos a - \sin x \sin a)$$

$$3 \cos x - \sin x = k \cos a \cos x - k \sin a \sin x$$

$$k \cos a = 3 \text{ and } k \sin a = 1$$

$$k = \sqrt{3^2 + 1^2}$$

$$k = \sqrt{10}$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{1}{3}$$

$$a = \tan^{-1}\left(\frac{1}{3}\right)$$

$$a = 0.32$$

$$\therefore 3 \cos x - \sin x = \sqrt{10} \cos(x + 0.32)$$

$$\text{where } 3 \cos x - \sin x = 2$$

$$\text{let } \sqrt{10} \cos(x + 0.32) = 2$$

$$\cos(x + 0.32) = \frac{2}{\sqrt{10}}$$

$$x + 0.32 = 0.89, 2\pi - 0.89$$

$$x + 0.32 = 0.89, 5.39$$

$$x = 0.57, 5.07$$

4. Let $2 \cos x - 3 \sin x = k \sin(x + a)$

$$2 \cos x - 3 \sin x = k(\sin x \cos a + \cos x \sin a)$$

$$2 \cos x - 3 \sin x = k \cos a \sin x + k \sin a \cos x$$

$$k \cos a = -3 \text{ and } k \sin a = 2$$

$$k = \sqrt{(-3)^2 + 2^2}$$

$$k = \sqrt{13}$$

$$\tan a = \frac{\sin a}{\cos a} = -\frac{2}{3}$$

$$\text{Acute } a = \tan^{-1}\left(\frac{2}{3}\right)$$

$$\text{Acute } a = 0.59 \therefore a = 2.55$$

$$\therefore 2 \cos x - 3 \sin x = \sqrt{13} \sin(x + 2.55)$$

$$\text{where } 2 \cos x - 3 \sin x = 3$$

$$\text{let } \sqrt{13} \sin(x + 2.55) = 3$$

$$\sin(x + 2.55) = \frac{3}{\sqrt{13}}$$

$$x + 2.55 = 0.98, \pi - 0.98$$

$$x + 2.55 = 0.98, 2.16$$

$$x = -1.57, -0.39$$

$$x = 4.71, 5.89$$

5. Let $\sqrt{5} \cos x - 3 \sin x = k \cos(x + a)^\circ$

$$\sqrt{5} \cos x - 3 \sin x = k(\cos x \cos a - \sin x \sin a)$$

$$\sqrt{5} \cos x - 3 \sin x = k \cos a \cos x - k \sin a \sin x$$

$$k \cos a = \sqrt{5} \text{ and } k \sin a = 3$$

$$k = \sqrt{5 + 9}$$

$$k = \sqrt{14}$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{3}{\sqrt{5}}$$

$$a = 53.3^\circ$$

$$\therefore \sqrt{5} \cos x - 3 \sin x = \sqrt{14} \cos(x + 53.3)^\circ$$

$$\text{Maximum value} = \sqrt{14}$$

$$\text{let } \sqrt{14} \cos(x + 53.3) = \sqrt{14}$$

$$\cos(x + 53.3) = 1$$

$$x + 53.3 = 0, 360$$

$$x = -53.3, 306.7$$

$$x = 306.7^\circ$$

6. Let $\sqrt{6} \sin x - 5 \cos x = k \sin(x + a)^\circ$

$$\sqrt{6} \sin x - 5 \cos x = k(\sin x \cos a + \cos x \sin a)$$

$$\sqrt{6} \sin x - 5 \cos x = k \cos a \sin x + k \sin a \cos x$$

$$k \cos a = \sqrt{6} \text{ and } k \sin a = 5$$

$$k = \sqrt{6 + 25}$$

$$k = \sqrt{31}$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{5}{\sqrt{6}}$$

$$a = 63.9^\circ$$

$$\therefore \sqrt{6} \sin x - 5 \cos x = \sqrt{31} \sin(x - 63.9)^\circ$$

$$\text{Minimum value} = -\sqrt{31}$$

$$\text{let } \sqrt{31} \sin(x - 63.9) = -\sqrt{31}$$

$$\sin(x - 63.9) = -1$$

$$x - 63.9 = 270$$

$$x = 333.9^\circ$$

7. Let $2 \sin x - 5 \cos x = k \cos(x - a)^\circ$

$$2 \sin x - 5 \cos x = k(\cos x \cos a + \sin x \sin a)$$

$$2 \sin x - 5 \cos x = k \cos a \cos x + k \sin a \sin x$$

$$k \cos a = -5 \text{ and } k \sin a = 2$$

$$k = \sqrt{5^2 + 2^2}$$

$$k = \sqrt{29}$$

$$\tan a = \frac{\sin a}{\cos a} = -\frac{2}{5}$$

$$\text{Acute } a = \tan^{-1}\left(\frac{2}{5}\right)$$

$$a = 180 - 21.8 = 158.2^\circ$$

$$\therefore 5 \cos x + 2 \sin x = \sqrt{29} \cos(x - 158.2)^\circ$$

$$\text{Minimum value} = -\sqrt{29}$$

$$\text{Let } \sqrt{29} \cos(x - 158.2) = -\sqrt{29}$$

$$\cos(x - 158.2) = -1$$

$$x - 158.2 = 270$$

$$x = 338.2^\circ$$

8. Let $\cos x - \sin x = k \cos(x + a)$

$$\cos x - \sin x = k(\cos x \cos a - \sin x \sin a)$$

$$\cos x - \sin x = k \cos a \cos x - k \sin a \sin x$$

$$k \cos a = 1 \text{ and } k \sin a = 1$$

$$k = \sqrt{1^2 + 1^2}$$

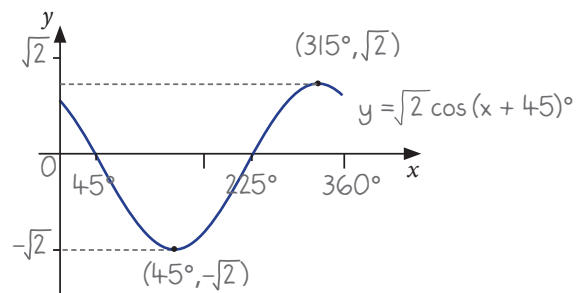
$$k = \sqrt{2}$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{1}{1}$$

$$a = \tan^{-1}\left(\frac{1}{1}\right)$$

$$a = 45$$

$$\therefore \cos x - \sin x = \sqrt{2} \cos(x + 45)^\circ$$



Roots on graph $(45, 0)$ and $(225, 0)$

Turning points $(45, -\sqrt{2})$ and $(315, \sqrt{2})$

Complete graph

9. Let $\sin x - \sqrt{3} \cos x = k \sin(x - a)^\circ$

$$\sin x - \sqrt{3} \cos x = k(\sin x \cos a - \cos x \sin a)$$

$$\sin x - \sqrt{3} \cos x = k \cos a \sin x - k \sin a \cos x$$

$$k \cos a = 1 \text{ and } k \sin a = \sqrt{3}$$

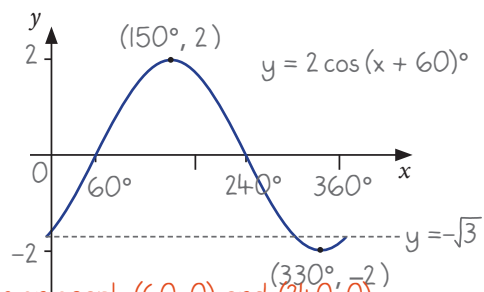
$$k = \sqrt{1 + 3}$$

$$k = 2$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{\sqrt{3}}{1}$$

$$a = 60^\circ$$

$$\therefore \sin x - \sqrt{3} \cos x = 2 \sin(x - 60)^\circ$$



Roots on graph $(60, 0)$ and $(240, 0)$

Turning points $(150, 2)$ and $(330, -2)$

Complete graph

10. Let $9\cos x - 2\sin x = k\cos(x + a)$

$$9\cos x - 2\sin x = k(\cos x \cos a - \sin x \sin a)$$

$$9\cos x - 2\sin x = k\cos a \cos x - k\sin a \sin x$$

$$k\cos a = 9 \text{ and } k\sin a = 2$$

$$k = \sqrt{9^2 + 2^2}$$

$$k = \sqrt{85}$$

$$\tan a = \frac{\sin a}{\cos a} = \frac{2}{9}$$

$$a = \tan^{-1}\left(\frac{2}{9}\right)$$

$$a = 12.5$$

$$\therefore 9\cos x - 2\sin x = \sqrt{85}\cos(x + 12.5)$$

$$\text{Maximum value} = 2\sqrt{85}$$

$$\text{let } 2\sqrt{85}\cos(x + 12.5) = 2\sqrt{85}$$

$$\cos(x + 12.5) = 1$$

$$x + 12.5 = 0, 360$$

$$x = -12.5, 347.5$$

$$x = 347.5^\circ$$

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Chapter 11

Calculus 1 – Differentiation

11D Exam Style Questions

1. $y = x^3 - 6x + 3$

$$\frac{dy}{dx} = 3x^2 - 6$$

$$\text{when } x = -1,$$

$$\frac{dy}{dx} = 3(-1)^2 - 6$$

$$= -3$$

$$\text{when } x = -1,$$

$$y = (-1)^3 - 6(-1) + 3$$

$$= 8$$

$$a = -1, b = 8, m = -3$$

$$y - b = m(x - a)$$

$$y - 8 = -3(x + 1)$$

$$y = -3x + 5$$

2. $-x^2 + x - 2$

$$= -(x^2 - x) - 2$$

$$= -\left(x - \frac{1}{2}\right)^2 - 2 - (-1)\left(\frac{1}{2}\right)^2$$

$$= -\left(x - \frac{1}{2}\right)^2 - \frac{8}{4} + \frac{1}{4}$$

$$= -\left(x - \frac{1}{2}\right)^2 - \frac{7}{4}$$

$$f'(x) = -x^2 + x - 2$$

$$-\left(x - \frac{1}{2}\right)^2 - \frac{7}{4} \text{ where } \left(x - \frac{1}{2}\right)^2 \geq 0 \text{ for all}$$

$$\text{values of } x$$

$$\text{Hence, } -\left(x - \frac{1}{2}\right)^2 - \frac{7}{4} < 0 \text{ for all value of } x$$

$$\therefore f'(x) < 0 \text{ for all } x \text{ so the function is strictly decreasing}$$

3. $A(x) = 24\pi x^2 + \frac{360}{x}$

$$A(x) = 24\pi x^2 + 360x^{-1}$$

$$A'(x) = 48\pi x - 360x^{-2}$$

$$A'(x) = 48\pi x - \frac{360}{x^2}$$

$$\text{For stationary points, } A'(x) = 0$$

$$0 = 48\pi x - \frac{360}{x^2}$$

$$48\pi x = \frac{360}{x^2}$$

$$48\pi x^3 = 360$$

$$x^3 = \frac{360}{48\pi}$$

$$x = \sqrt[3]{\frac{360}{48\pi}}$$

$$x = 1.336... = 1.3$$

x	1.3 ⁺	1.3	1.3
A'(x)	-	0	+
slope	↘	—	↗

$$\therefore \text{Minimum when } x = 1.3 \text{ cm}$$

4. $s(t) = \frac{1}{3t^2}$

$$s(t) = \frac{1}{3}t^{-2}$$

$$s'(t) = -\frac{2}{3}t^{-3}$$

$$s'(t) = -\frac{2}{3t^3}$$

$$s'(4) = -\frac{2}{3(4)^3} = -\frac{1}{96}$$

5. $y = 2x^3 + 4x^2 - 8x - 1$

$$\frac{dy}{dx} = 6x^2 + 8x - 8$$

For stationary points, $\frac{dy}{dx} = 0$

$$6x^2 + 8x - 8 = 0$$

$$2(3x - 2)(x + 2) = 0$$

$$3x - 2 = 0 \text{ or } x + 2 = 0$$

$$x = \frac{2}{3} \text{ or } x = -2$$

When $x = -2$, $y = 15$

When $x = \frac{2}{3}$, $y = -\frac{101}{27}$

$\frac{dy}{dx}$	-	0	+
-----------------	---	---	---

slope			
-------	---	---	---

x	-2^+	-2	-2^-
-----	--------	------	--------

$\frac{dy}{dx}$	+	0	-
-----------------	---	---	---

slope			
-------	--	--	--

Max TP at $(-2, 15)$ and Min TP at $(\frac{2}{3}, -\frac{101}{27})$

6. $f(x) = 4x^3 - 12x$

$$f'(x) = 12x^2 - 12$$

$$f'(x) = 12(x^2 - 1) = 12(x - 1)(x + 1)$$

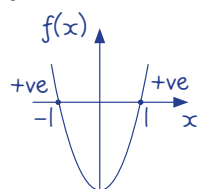
For increasing functions, $f'(x) > 0$,

$$12(x - 1)(x + 1) > 0$$

Let $f(x) = 0$,

$$12(x - 1)(x + 1) = 0$$

$f(x) = 0$ when $x = -1$ and $x = 1$

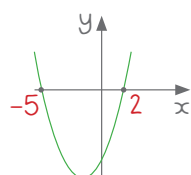


increasing when $x < -1$ and $x > 1$

7. $y = x^{\frac{4}{3}} - 4x^{-2}$

$$\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}} + 8x^{-3}$$

8. graph of $f'(x)$



Points $(-5, 0)$ and $(2, 0)$ marked

Correct parabola shape

9. $f(x) = 2x^4 - 3x^2 + 6$

$$f'(x) = 8x^3 - 6x^2$$

$$f'(-2) = 8(-2)^3 - 6(-2)^2 = -52$$

As $f'(x) < 0$, the function is strictly decreasing at $x = -2$

10. $y = 7x - 42x^{\frac{1}{2}}$

$$\frac{dy}{dx} = 7 - 42x^{-\frac{1}{2}} + 7$$

$$\frac{dy}{dx} = 7 - \frac{21}{\sqrt{x}}$$

For stationary points, $\frac{dy}{dx} = 0$

$$0 = 7 - \frac{21}{\sqrt{x}}$$

$$x = 9$$

When,

$$x = 9, y = -63$$

$$x = 1, y = -35$$

$$x = 16, y = -56$$

$\therefore$ maximum value is -35 at $x = 1$

minimum value is -63 at $x = 9$

Chapter 12

Calculus 2 – Integration

12D Exam Style Questions

1. $\int (10x^4 - 8\sqrt{x}) dx$

$$= \frac{10x^5}{5} - \frac{8x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2x^5 - 16x^{\frac{1}{2}} + C$$

2. $\frac{dy}{dx} = 16x^3 - 5$

$$y = \int (16x^3 - 5) dx$$

$$y = \frac{16x^4}{4} - 5x + C$$

when $x = -2$ and $y = 24$,

$$24 = 4(-2)^4 - 5(-2) + C$$

$$24 = 64 + 10 + C$$

$$C = -50 \therefore y = 4x^4 - 5x - 50$$

$$\begin{aligned}
 3. \quad & \int_1^2 \frac{1}{4}x^{-3} dx \\
 &= \left[\frac{x^{-2}}{4(-2)} \right]_1^2 \\
 &= \left[-\frac{1}{8x^2} \right]_1^2 \\
 &= \left[-\frac{1}{8x^2} \right]_1^2 \\
 &= \left(-\frac{1}{8(2)^2} \right) - \left(-\frac{1}{8(1)^2} \right) \\
 &= \left(-\frac{1}{32} \right) - \left(-\frac{1}{8} \right) \\
 &= \frac{3}{32}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \int \frac{3x^2 - 12}{x^4} dx \\
 &= \int x^{-4}(3x^2 - 12) dx \\
 &= \int (3x^{-2} - 12x^{-4}) dx \\
 &= \frac{3x^{-1}}{-1} - \frac{12x^{-3}}{-3} + C \\
 &= -3x^{-1} + 4x^{-3} + C
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & V = \int \left(\frac{1}{4}t + k \right) dt \\
 & V = \frac{t^2}{4(2)} + kt + C \\
 & \text{sub values into equation above and solve} \\
 & \text{simultaneously}
 \end{aligned}$$

$$10 = 2 + 4k + c \rightarrow 4k + c = 8$$

$$60 = 8 + 8k + c \rightarrow 8k + c = 52$$

$$k = 11 \text{ and } c = -36$$

$$V = \frac{1}{8}t^2 + 11t - 36$$

$$\begin{aligned}
 6. \quad & \int_5^7 (x^2 - 8x + 19) dx \\
 &= \left[\frac{x^3}{3} - \frac{8x^2}{2} + 19x \right]_5^7 \\
 &= \left[\frac{x^3}{3} - 4x^2 + 19x \right]_5^7 \\
 &= \left(\frac{7^3}{3} - 4(7)^2 + 19(7) \right) - \left(\frac{5^3}{3} - 4(5)^2 + 19(5) \right) \\
 &= \frac{44}{3} \therefore \text{Area} = \frac{44}{3} \text{ units}^2
 \end{aligned}$$

7. Curves intersect when $y = y$

$$\begin{aligned}
 \therefore x^3 - 2x + 1 &= x - 1 \\
 x^3 - 3x + 2 &= 0 \\
 (x - 1)(x^2 + x - 2) &= 0
 \end{aligned}$$

$$(x - 1)(x - 1)(x + 2) = 0$$

$$x = 1 \text{ and } x = -2$$

$$\begin{aligned}
 \text{Area} &= \int_{-2}^1 (\text{upper curve}) - (\text{lower curve}) dx \\
 &= \int_{-2}^1 (x^3 - 2x + 1) - (x - 1) dx \\
 &= \int_{-2}^1 (x^3 - 3x + 2) dx
 \end{aligned}$$

$$\begin{aligned}
 &= \left[\frac{x^4}{4} - \frac{3x^2}{2} + 2x \right]_{-2}^1 \\
 &= \left(\frac{1^4}{4} - \frac{3(1)^2}{2} + 2(1) \right) - \left(\frac{(-2)^4}{4} - \frac{3(-2)^2}{2} + 2(-2) \right) \\
 &= \frac{27}{4} \therefore \text{Area} = \frac{27}{4} \text{ units}^2
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \text{Area} = \int_2^5 (\text{upper curve}) - (\text{lower curve}) dx \\
 &= \int_2^5 (-3x^2 + 20x - 30) - (2x^2 - 15x + 20) dx \\
 &= \int_2^5 (-5x^2 + 35x - 50) dx \\
 &= \left[-\frac{5x^3}{3} + \frac{35x^2}{2} - 50x \right]_2^5 \\
 &= \left(-\frac{5(5)^3}{3} + \frac{35(5)^2}{2} - 50(5) \right) - \left(-\frac{5(2)^3}{3} + \frac{35(2)^2}{2} - 50(2) \right) \\
 &= \frac{45}{2} \therefore \text{Area} = \frac{45}{2} \text{ units}^2
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \int (x^{\frac{1}{2}} + 5x^3) dx \\
 &= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{5x^4}{4} + C \\
 &= 2x^{\frac{3}{2}} + \frac{5x^4}{4} + C
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & f(x) = \int (x^{-2} + 3) dx \\
 & f(x) = \frac{x^{-1}}{-1} + 3x + C \\
 & f(x) = -\frac{1}{x} + 3x + C \\
 & f(1) = 12
 \end{aligned}$$

$$12 = -\frac{1}{1} + 3(1) + C$$

$$C = 10$$

$$f(x) = -\frac{1}{x} + 3x + 10$$

$$f(2) = -\frac{1}{2} + 3(2) + 10$$

$$\therefore f(2) = \frac{31}{2}$$

Chapter 13

Calculus 3 – Further Calculus

13D Exam Style Questions

1. $f'(x) = 4 \cos 2x - \sin x$

Let $y = f(x)$

$$y = \int (4 \cos 2x - \sin x) dx$$

$$y = 2 \sin 2x + \cos x + C$$

$$y = 2 \text{ when } x = \pi$$

$$2 = 2 \sin 2\pi + \cos \pi + C$$

$$\sin 2\pi = 0, \cos \pi = -1$$

$$2 = 2(0) + (-1) + C$$

$$C = 3, \therefore y = 2 \sin 2x + \cos x + 3$$

2. $\int_{\frac{\pi}{3}}^{\pi} (4 \cos x - 2 \sin x) dx$

$$= [4 \sin x + 2 \cos x]_{\frac{\pi}{3}}^{\pi}$$

$$= (4 \sin \pi + 2 \cos \pi) - (4 \sin \frac{\pi}{3} + 2 \cos \frac{\pi}{3})$$

$$= (4(0) + 2(-1)) - (2\sqrt{3} + 1)$$

$$= -3 - 2\sqrt{3}$$

3. $\int 8 \sin(2x + \frac{\pi}{6}) dx$

$$= -\frac{8 \cos(2x + \frac{\pi}{6})}{2} + C$$

$$= -4 \cos(2x + \frac{\pi}{6}) + C$$

4. $\int_2^3 (5 - 2x)^{-\frac{1}{3}} dx$

$$= \left[\frac{(5 - 2x)^{\frac{2}{3}}}{-\frac{4}{3}} \right]_2^3$$

$$= \left(\frac{-3(5 - 2(3))^{\frac{2}{3}}}{4} \right) - \left(\frac{-3(5 - 2(2))^{\frac{2}{3}}}{4} \right)$$

$$= 0$$

5. $f(x) = (x^3 + 1)^4$

$$f'(x) = 4(x^3 + 1)^3 \times 3x^2$$

$$f'(x) = 12x^2(x^3 + 1)^3$$

$$f'(1) = 12(1)^2((1)^3 + 1)^3$$

$$f'(1) = 96$$

6. $f(x) = 2 \cos(4x - \frac{\pi}{2})$

$$f'(x) = -2 \sin(4x - \frac{\pi}{2}) \times 4$$

$$f'(x) = -8 \sin(4x - \frac{\pi}{2})$$

$$f'(\frac{\pi}{4}) = -8 \sin(\frac{4\pi}{4} - \frac{\pi}{2})$$

$$= -8 \sin(\frac{\pi}{2})$$

$$= -8$$

7. $\int 4(2x - 3)^{-\frac{1}{2}} dx$

$$= \frac{4(2x - 3)^{\frac{1}{2}}}{\frac{1}{2} \times 2} + C$$

$$= 4(2x - 3)^{\frac{1}{2}} + C$$

8. $f(x) = 5 \cos^2 x$

$$f(x) = 5 (\cos x)^2$$

$$f'(x) = 10 \cos x \times -\sin x$$

$$f'(x) = -10 \sin x \cos x$$

$$f'(\frac{2\pi}{3}) = -10 \sin \frac{2\pi}{3} \cos \frac{2\pi}{3}$$

$$= -10(\frac{\sqrt{3}}{2})(\frac{1}{2})$$

$$= -\frac{5\sqrt{3}}{2}$$

9. $\int_0^a (2x + 1)^3 dx = 10$

$$\left[\frac{(2x + 1)^4}{4 \times 2} \right]_0^a = 10$$

$$\frac{(2a + 1)^4}{8} - \frac{1}{8} = 10$$

$$(2a + 1)^4 - 1 = 80$$

$$(2a + 1)^4 = 81$$

$$(2a + 1) = 3$$

$$2a = 2$$

$$a = 1$$

$$\begin{aligned}
 10. \int_0^{13} (2x+1)^{-\frac{1}{3}} dx &= \left[\frac{(2x+1)^{\frac{2}{3}}}{\frac{2}{3} \times 2} \right]_0^{13} \\
 &= \left[\frac{3(2x+1)^{\frac{2}{3}}}{4} \right]_0^{13} \\
 &= \frac{3(27)^{\frac{2}{3}}}{4} - \frac{3(1)^{\frac{2}{3}}}{4} \\
 &= \frac{3(9)}{4} - \frac{3(1)}{4} \\
 &= 6
 \end{aligned}$$

Chapter 14

Vectors

14D Exam Style Questions

$$\begin{aligned}
 1. \quad \vec{AB} &= \vec{b} - \vec{a} = \begin{pmatrix} 6 \\ -2 \\ -3 \end{pmatrix} - \begin{pmatrix} 6 \\ -11 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \\ -6 \end{pmatrix} \\
 \vec{BC} &= \vec{c} - \vec{b} = \begin{pmatrix} 6 \\ 1 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ -21 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}
 \end{aligned}$$

Since $\vec{AB} = 3\vec{BC}$, the vectors are parallel. Point B is common. $\therefore$ Points A, B and C are collinear.

B divides AC in a ratio 3:1

2. Difference in coordinates $x = 0$, $y = -20$, $z = 15$

Point B is $\frac{1}{5}$ along AC

$$B: \frac{1}{5} \times 0 = 0, \frac{1}{5} \times -20 = -4, \frac{1}{5} \times 15 = 3$$

Coordinate of B: $(1 + 0, 4 + (-4), (-11) + 3)$

$$B = (1, 0, -8)$$

3. For perpendicular vectors, $\mathbf{u} \cdot \mathbf{v} = 0$

$$2\mathbf{a} \times \mathbf{a} + (-10) \times \mathbf{a} + 3 \times (-4) = 0$$

$$2a^2 - 10a - 12 = 0$$

$$2(a - 6)(a + 1) = 0$$

$$a = -1, 6$$

$$\begin{aligned}
 4. \quad \vec{ED} &= \mathbf{d} - \mathbf{e} = \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ -1 \end{pmatrix} \\
 \vec{EF} &= \mathbf{f} - \mathbf{e} = \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ -11 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}
 \end{aligned}$$

$$|\vec{ED}| = \sqrt{6^2 + 0^2 + (-1)^2} = \sqrt{37}$$

$$|\vec{EF}| = \sqrt{5^2 + 3^2 + (-3)^2} = \sqrt{43}$$

$$\vec{ED} \cdot \vec{EF} = 6 \times 5 + 0 \times 3 + (-1) \times (-3) = 33$$

$$\cos \theta = \frac{33}{\sqrt{37}\sqrt{43}}$$

$$\theta = 34.2^\circ$$

$$\begin{aligned}
 5. \quad \vec{PD} &= \vec{FD} - \vec{FP} \\
 \vec{PD} &= \vec{FB} + \vec{BA} + \vec{AD} + \vec{FP} \\
 \vec{PD} &= -\frac{1}{2}\vec{AB} - \vec{AB} + \vec{AD} + \vec{FP} \\
 \vec{PD} &= -\frac{1}{2}\begin{pmatrix} 6 \\ -8 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ -8 \\ 4 \end{pmatrix} + \begin{pmatrix} 7 \\ -11 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 7 \end{pmatrix} \\
 \vec{PD} &= \begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad 2\mathbf{a} + \mathbf{b} &= \begin{pmatrix} 2p \\ 2 \\ 14 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \\ -13 \end{pmatrix} = \begin{pmatrix} 2p+4 \\ 2 \\ 1 \end{pmatrix} \\
 |2\mathbf{a} + \mathbf{b}| &= \sqrt{(2p+4)^2 + 2^2 + 1^2} = 3
 \end{aligned}$$

$$4p^2 + 16p + 12 = 0$$

$$4(p+3)(p+1) = 0$$

$$p = -1, -3$$

$$\begin{aligned}
 7. \quad \mathbf{u} \cdot (\mathbf{u} + \mathbf{v}) &= \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} \\
 &= \sqrt{2} \times \sqrt{2} \cos 0 + \sqrt{2} \times \sqrt{2} \cos 60 \\
 &= 2 + 2 \times \frac{1}{2} = 3
 \end{aligned}$$

8. Difference in coordinates from A to B:

$$x = -3, y = 1, z = -2$$

As, B to C is 3 times from A to B, distances are,

$$x = -9, y = 3, z = -6$$

Coordinate of C: $(3 + (-9), (-1) + 3, 1 + (-6))$

$$C = (-6, 2, -5)$$

$$\begin{aligned}
 9. \quad p(q+r) &= p.q + p.r \\
 &= 4 \times 4 \cos 60 + 4 \times 4 \cos 90 \\
 &= 16 \times \frac{1}{2} + 16 \times 0 = 8
 \end{aligned}$$

$$10. \quad 3u = 4v$$

$$\begin{pmatrix} 6p \\ 12 \\ -24 \end{pmatrix} = \begin{pmatrix} 4p - 40 \\ 12 \\ -24 \end{pmatrix}$$

$$6p = 4p - 40$$

$$p = -20$$

Chapter 15

Logarithmic & Exponential Functions

15D Exam Style Questions

$$1. \quad 2 \log_6 3 + 2 \log_6 2$$

$$= \log_6 3^2 + \log_6 2^2$$

$$= \log_6 (3^2 \times 2^2)$$

$$= \log_6 (36)$$

$$= \log_6 (36)$$

$$= 2$$

$$2. \quad \log_2 18x - \log_2 6 = \log_6 36$$

$$\log_2 \left(\frac{18x}{6} \right) = 2$$

$$\log_2 (3x) = 2$$

$$3x = 2^2$$

$$x = \frac{4}{3}$$

$$3. \quad \log_a 18 - 2 \log_a 3 = \frac{1}{3}$$

$$\log_a 18 - \log_a 3^2 = \frac{1}{3}$$

$$\log_a \left(\frac{18}{9} \right) = \frac{1}{3}$$

$$a^{\frac{1}{3}} = 2$$

$$\sqrt[3]{a} = 2$$

$$a = 8$$

$$4. \quad 2 \log_6 (x+8) - \log_6 x = 2$$

$$\log_6 (x+8)^2 - \log_6 x = 2$$

$$\log_6 (x^2 + 16x + 64) - \log_6 x = 2$$

$$\log_6 \left(\frac{x^2 + 16x + 64}{x} \right) = 2$$

$$\frac{x^2 + 16x + 64}{x} = 6^2$$

$$x^2 + 16x + 64 = 36$$

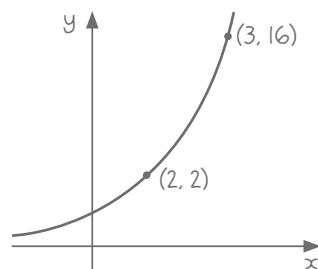
$$x^2 - 20x + 64 = 0$$

$$(x-4)(x-16) = 0$$

$$x = 4, 16$$

5. vertical scaling by a factor of 2 identifiable from graph

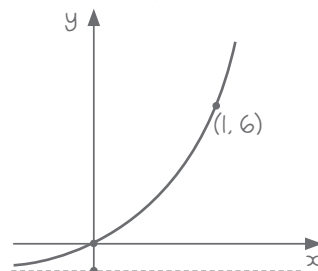
horizontal translation of '+2' units identifiable from graph



6. graph reflected in $y = x$

vertical translation of '-1' unit following a reflection in $y = x$ identifiable from graph

sketch of required function



$$7. \quad V(t) = 150e^{0.28t}$$

$$V(24) = 150e^{0.28(24)}$$

$$= 124\,322.626\dots$$

$$= 124\,000$$

$$8. \quad V(t) = 150e^{0.28t}$$

$$200\,000 = 150e^{0.28t}$$

$$\frac{200\,000}{150} = e^{0.28t}$$

$$\log_e \frac{200\,000}{150} = \log_e e^{0.28t}$$

$$\log_e \frac{200\,000}{150} = 0.28t \log_e e = 1$$

$$t = 25.70 \text{ hours}$$

9. $y = kx^n$

$$\log_3 y = \log_3 kx^n \quad 1$$

$$\log_3 y = n \log_3 x + \log_3 k \quad 1$$

$$n = \text{gradient}$$

$$n = \frac{\text{vertical}}{\text{horizontal}}$$

$$n = \frac{6}{3} = 2 \quad 1$$

$$\log_3 k = -2 \quad 1$$

$$k = 3^{-2}$$

$$k = \frac{1}{9} \quad 1$$

10. $y = ka^x$

$$\log_{16} y = \log_{16} ka^x \quad 1$$

$$\log_{16} y = x \log_{16} a + \log_{16} k \quad 1$$

$$\log_{16} a = \text{gradient}$$

$$\text{gradient} = \frac{\text{vertical}}{\text{horizontal}}$$

$$\text{gradient} = \frac{2}{8} = \frac{1}{4}$$

$$\log_{16} a = \frac{1}{4} \quad 1$$

$$\log_{16} k = \frac{1}{2} \quad 1$$

$$a = 2, k = 4 \quad 1$$